Name: _____

Matricola: _____ email: _____

ECONOMETRICS - 09-20-2017 - Time: 2 h 30 m

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. A "CAN'T SAY" answer with no motivations will be considered wrong.

   (a) A non-zero column vector has full rank.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (b) A non-zero row vector has full rank.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (c) If $P_X = X(X'X)^{-1}X'$ and $M_X = I - P_X$, then the product $M_X P_X P_X P_X M_X M_X P_X P_X P_X M_X$ yields a zero matrix.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (d) In a linear model, if the sum of squared residuals is zero, the the R^2 index is also zero.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (e) The null hypothesis for the Godfrey test is the presence of autocorrelation.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

2. Consider the model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, where both y_i and x_i are dummy variables. Let n be the sample size, and

$$p_y = \frac{1}{n}\sum_{i=1}^{n} y_i \quad p_x = \frac{1}{n}\sum_{i=1}^{n} x_i \quad c = \frac{1}{n}\sum_{i=1}^{n} x_i y_i.$$

(a) Write the matrix $X'X$ as a function of n and p_x.

$$X'X = \begin{bmatrix} & \\ & \end{bmatrix}$$

(b) Write the vector $\hat{\beta}$ as a function of p_x, p_y and c.

$$\hat{\beta} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

(c) Write the scalar $\hat{\sigma}^2$ as a function of n, p_x, p_y and c.

$$\hat{\sigma}^2 =$$

(d) What happens if $c = p_x \cdot p_y$? What is your statistical intuition for the result?

(e) Suppose that $n = 100$, $p_x = 0.57$, $p_y = 0.8$ and $c = 0.54$. Test the hypothesis $H_0 : \beta_1 = 0$

Test type:	_____	Distribution:	_____	Test statistic:	_____
Decision:	REJECT ◯		DON'T REJECT ◯		

3. Table 1 contains a model where the dependent variable y is the logarithm of the number of patents that a firm holds; the sample contains 781 large European manufacturing firms. The independent variables are as follows:

Variable	**Description**
x	Firm size (as logarithm of operating revenue)
UK	dummy; 1 if firm is British
Germany	dummy; 1 if firm is German
France	dummy; 1 if firm is French
Electronic	dummy; 1 if firm belongs to the "Computer, electronic and optical products" industry
Machinery	dummy; 1 if firm belongs to the "Machinery and equipment" industry
Chemical	dummy; 1 if firm belongs to the "Chemicals and chemical products" industry
Food	dummy; 1 if firm belongs to the "Food products" industry
Pharma	dummy; 1 if firm belongs to the "Basic pharmaceutical products and pharmaceutical preparations" industry

Descriptive statistics are given in Table 2.

Table 1: OLS model

Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	4.6059	1.3784	3.3416	0.0009
x	−0.9496	0.2480	−3.8298	0.0001
x^2	0.0591	0.0109	5.4155	0.0000
UK	−1.0109	0.1650	−6.1277	0.0000
Germany	1.1456	0.2109	5.4318	0.0000
France	0.1822	0.1709	1.0656	0.2869
Electronic	0.9055	0.1936	4.6782	0.0000
Machinery	0.9723	0.2226	4.3677	0.0000
Chemical	0.7943	0.2619	3.0333	0.0025
Food	−0.9496	0.1793	−5.2972	0.0000
Pharma	0.8903	0.2823	3.1542	0.0017

Mean dependent var	2.147448	S.D. dependent var	2.377860
Sum squared resid	2512.467	S.E. of regression	1.806362
R^2	0.430317	Adjusted R^2	0.422919
$F(10, 770)$	47.61482	P-value(F)	6.51e–74

Provide an economic interpretation of the estimates presented in Table 1 in a separate sheet.

Table 2: Descriptive statistics

Variable	Mean	Median	S.D.	Min	Max
y	2.147	1.386	2.378	0.000	12.16
x	11.69	11.57	2.299	6.946	19.26
UK	0.2548	0.000	0.4360	0	1
Germany	0.1626	0.000	0.3692	0	1
France	0.1536	0.000	0.3608	0	1
Electronic	0.1370	0.000	0.3441	0	1
Machinery	0.1204	0.000	0.3256	0	1
Chemical	0.08323	0.000	0.2764	0	1
Food	0.08067	0.000	0.2725	0	1
Pharma	0.06786	0.000	0.2517	0	1