Name: _____

Matricola: _____ email: _____

ECONOMETRICS - 12-07-2017 - Time: 2 h 30 m

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. A "CAN'T SAY" answer with no motivations will be considered wrong.

   (a) A non-zero column vector has full rank.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (b) A non-zero row vector has full rank.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (c) If $P_X = X(X'X)^{-1}X'$ and $M_X = I - P_X$, then the product $M_X P_X P_X M_X P_X P_X M_X$ yields a non-zero matrix.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (d) Consider the linear model $y_i = \beta_0 + \beta_1 d_i + \beta_2 d_i^2 + \varepsilon_i$, where d_i is a dummy variable. This model cannot be estimated by OLS.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (e) The null hypothesis for White's test is the presence of heteroskedasticity.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

2. Consider the model $y_i = \beta_0 + \beta_1 d_i + \varepsilon_i$ where y_i is the mark in econometrics obtained by individual i and d_i is a dummy variable taking value 1 if individual i has blue eyes. Suppose that we have a sample of $n = 100$ individuals, 25 of whom have blue eyes. The average mark for the whole sample is 25, while the average mark for people with blue eyes is 24. Furthermore, $\sum_{i=1}^{n} y_i^2 = 63549$.

   (a) calculate the matrices $\mathbf{X}'\mathbf{X}$ and $\mathbf{X}'\mathbf{y}$;

$$\mathbf{X}'\mathbf{X} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} \qquad \mathbf{X}'\mathbf{y} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

   (b) calculate the OLS statistic;

$$\hat{\beta} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

   (c) calculate the covariance matrix for $\hat{\beta}$

$$\hat{V} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

   (d) calculate the R^2 index: $R^2 =$ _____

   (e) test the hypothesis that blue-eyed people don't get penalised:

Test type:	_____	Distribution:	_____	Test statistic:	_____
Decision:	REJECT	◯	DON'T REJECT	◯	

3. Table 1 contains a model with weekly observations from 1996-05-10 to 2017-06-30, where the dependent variable y_t is the logarithm of the price of Diesel fuel; the independent variable x_t the log of oil price (WTI, \$ per barrel).

   (a) write the coefficients for the ECM form in the blank spaces:

$$\begin{aligned} \Delta y_t \;=\; & _____ + _____ \Delta x_t + \\ & + _____ \Delta y_{t-1} + _____ \Delta y_{t-2} + \\ & -(_____) \cdot [\, y_{t-1} - _____ x_{t-1} \,] + \varepsilon_t \end{aligned}$$

   (b) Calculate the short-run elasticity:

$$\delta_0 = _____$$

   (c) Calculate the long-run elasticity:

$$\sum_{j=0}^{\infty} \delta_j = _____$$

Table 1: OLS model

OLS, using observations 1996-05-10–2017-06-30 ($T = 1104$)

	Coefficient	Std. Error	t-ratio	p-value
const	-0.347	0.040	-8.7008	0.0000
x_t	0.597	0.024	24.3938	0.0000
x_{t-1}	-0.497	0.027	-18.5405	0.0000
y_{t-1}	1.160	0.026	43.9362	0.0000
y_{t-2}	-0.384	0.038	-10.0602	0.0000
y_{t-3}	0.119	0.024	4.9976	0.0000

Mean dependent var	0.362349	S.D. dependent var	0.595308
Sum squared resid	1.316202	S.E. of regression	0.034623
R^2	0.996633	Adjusted R^2	0.996618
$F(5, 1098)$	64998.58	P-value(F)	0.000000

LM test for autocorrelation up to order 4 – Test statistic: LMF = 1.84785
with p-value = $P(F(4, 1094) > 1.84785) = 0.11743$

(d) Add any further comment on the results you deem appropriate in the space below: