Name: _____

Matricola: _____ email: _____

ECONOMETRICS - 17-02-2017 - Time: 2 h 30'

1. Say if the following statements are unambiguously true (TRUE), unambiguously false (FALSE) or impossible to classify the way they are stated (CAN'T SAY). Write the motivations to your answers **only** in the space provided. A "CAN'T SAY" answer with no motivations will be considered wrong.

   (a) Projection matrices are not always square.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (b) If $E(y|x) = 0$, then y and x are independent.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (c) If $E(y|x) = x^2$, then y and x are not independent.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (d) If $X_n \xrightarrow{\text{p}} 2$, then the probability that $3 \leq X_n \leq 4$ tends to 0 as $n \to \infty$.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

   (e) If $X_n \xrightarrow{\text{d}} N(2, 1)$, then the probability that $3 \leq X_n \leq 4$ tends to 0 as $n \to \infty$.

   TRUE ◯ FALSE ◯ CAN'T SAY ◯

2. Using data on 7844 households[1] from the *Survey of Household Income and Wealth* (Bank of Italy, 2014), the following model was estimated:

$$c_i = \beta_0 + \beta_1 y_i + \beta_2 n_i + \beta_3 y_i^2 + \beta_4 y_i \cdot n_i + \beta_5 k_i + \beta_6 ne_i + \beta_6 ce_i + \beta_7 so_i + \beta_7 is_i \tag{1}$$

where

Variable	**Description**
c_i	log of consumption for nondurable goods
y_i	log of disposable income
n_i	log of household size
k_i	"kids" dummy (equals 1 if at least one household member was born after 1996)
ne_i, ce_i, so_i, is_i	Geographical dummies: North-East, Centre, South, islands

Dependent variable: c
Heteroskedasticity-robust standard errors, variant HC1

	Coefficient	Std. Error	t-ratio	p-value
const	9.1366	1.0971	8.328	9.65e-17
y_i	−0.52478	0.22311	−2.352	0.0187
n_i	0.5895	0.15902	3.707	0.000211
y_i^2	0.058574	0.011347	5.162	2.5e-07
$y_i \cdot n_i$	−0.053792	0.015846	−3.395	0.00069
k_i	0.03618	0.0094117	3.844	0.000122
ne_i	−0.062274	0.0088416	−7.043	2.04e-12
ce_i	−0.0034313	0.0085346	−0.402	0.688
so_i	−0.053791	0.0093237	−5.769	8.26e-09
is_i	−0.031413	0.010075	−3.118	0.00183

Mean dependent var	9.896388	S.D. dependent var	0.478559
Sum squared resid	528.0931	S.E. of regression	0.259635
R^2	0.705994	Adjusted R^2	0.705656
$F(9, 7834)$	1439.327	P-value(F)	0.000000

(a) Household A (2 adults, 1 kid) lives in Rome, and their disposable income is 25000 €/year; estimate their annual expenditure in nondurable goods:

$$\hat{C}_A = \underline{\hspace{6cm}}$$

(b) Household B (2 adults, no kids) lives in Naples, and their disposable income is 40000 €/year; estimate their annual expenditure in nondurable goods:

$$\hat{C}_B = \underline{\hspace{6cm}}$$

(c) Comment on the estimated coefficients for the "kids" and geographical dummies.

[1] The complete survey includes 8156 housholds; we excluded the households that reported a per capita yearly income lower than 3000 €.

(d) Write down an analytical expression (that is, a formula containing the β_i coefficients plus any other terms that may be needed) for the elasticity of consumption with respect to income:

$$\epsilon = \frac{\partial C_i / C_i}{\partial Y_i / Y_i} = \frac{\partial \ln C_i}{\partial \ln Y_i} = \frac{\partial c_i}{\partial y_i} = \underline{\hspace{6cm}}$$

(e) Estimate the income elasticity for households A and B:

$$\epsilon_A = \underline{\hspace{3cm}} \qquad \epsilon_A = \underline{\hspace{3cm}}$$

(f) This is the covariance matrix for the OLS estimator of the parameters $[\beta_3, \beta_4]$:

$$\hat{V} = \frac{1}{10000} \begin{bmatrix} 1.287 & -1.192 \\ -1.192 & 2.511 \end{bmatrix}$$

Test the joint hypothesis $\beta_3 = 0, \beta_4 = 0$ and discuss its economic implication for the income elasticity ϵ.

Test type:	_____	Distribution:	_____	Test statistic:	_____
Decision:	REJECT	◯	DON'T REJECT	◯	

_____

_____

_____

_____

_____

_____

Figure 1: Geography reminder

Nota Bene:

$$\begin{aligned} \ln 2 &= 0.69315 \\ \ln 3 &= 1.0986 \\ \ln 25000 &= 10.1266 \\ \ln 40000 &= 10.5966 \end{aligned}$$